EXERCISE – V

HINTS & SOLUTIONS

Sol.1 $\dfrac{2}{r_2} = \dfrac{1}{r_1} + \dfrac{1}{r_3}$ & Δ = 24 sq. cm

& 2S = 24 cm

s = 12

$\Rightarrow \dfrac{2(s-b)}{\Delta} = \dfrac{s-a+s-c}{\Delta}$

$\Rightarrow$ 2 (s – b) = 2s – a – c

$\Rightarrow$ 2b = a + c $\Rightarrow$ a, b, c in A.P.

a + b + c = 24

b + 2b = 24 $\Rightarrow$ 3b = 24 $\Rightarrow$ b = 8

$\Rightarrow$ a + c =16

Δ^2 = s(s – a) (s – b) (s – c)

(24) . (24) = 12 (12 – a) (12 – 8) (12 – c)

$\Rightarrow$ 12 = 144 – 12 (a + c) + ac

$\Rightarrow$ 12 = 12 (12 – 16) + ac

$\Rightarrow$ ac = 60

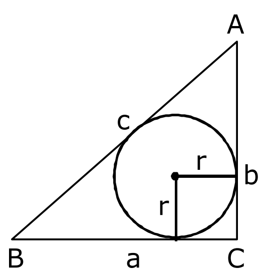
$\therefore$ a + $\dfrac{60}{a}$ = 16 $\Rightarrow$ $a^2 - 16a + 60 = 0$

$\Rightarrow$ (a – 6) (a – 10) = 0

$\Rightarrow$ a = 6, a = 10

$\therefore$ a = 6, b = 8, c = 10

Sol.2 **(a) A**



c = 2R

r = (s – c) tan $\dfrac{C}{2}$ = (s – c) tan $\dfrac{\pi}{4}$ = (s – c)

2r = 2s – 2c

$\Rightarrow$ 2(r + R) = 2s – 2c + c

2(r + R) = 2s – c

= a + b + c – c

= a + b

(b) B

$2\,ac \sin\left(\dfrac{A-B+C}{2}\right)$ {A + C = π – B}

$= 2ac \sin\left(\dfrac{\pi - 2B}{2}\right)$

$= 2ac \sin\left(\dfrac{\pi}{2} - B\right)$

$= 2ac \cos B = c^2 + a^2 - b^2$

Sol.3 $\dfrac{r_1}{r - r_1} = \cot \dfrac{A}{2}$

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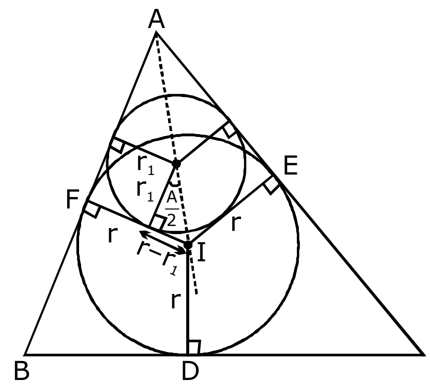
$\dfrac{r_2}{r - r_2} = \cot \dfrac{B}{2}$



$\dfrac{r_3}{r - r_3} = \cot \dfrac{C}{2}$

$\Sigma \cot \dfrac{A}{2} = \pi \cot \dfrac{A}{2}$

$\dfrac{r_1}{r-r_1} + \dfrac{r_2}{r-r_2} + \dfrac{r_3}{r-r_3} = \dfrac{r_1.r_2.r_3}{(r-r_1)(r-r_2)(r-r_3)}$

Sol.4 Let assume that given inequality is true

$\Delta \le \dfrac{1}{4}\sqrt{(a+b+c)\,abc}$

$16\Delta^2 \le (a + b + c)abc$

$\Rightarrow \dfrac{16\Delta^2}{2s} \le abc$

$\Rightarrow 8\Delta r \le abc$

$\Rightarrow 2r \le \dfrac{abc}{4\Delta}$

$\Rightarrow 2r \le R$

$\Rightarrow 2.4 \sin \dfrac{A}{2} \sin \dfrac{B}{2} \sin \dfrac{C}{2} \le 1$

$\Rightarrow \sin \dfrac{A}{2} \sin \dfrac{B}{2} \sin \dfrac{C}{2} \le \dfrac{1}{8}$ is True

If A = B = C then equality holds true

$\Rightarrow$ a = b = c

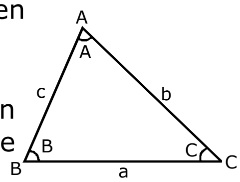
Sol.5 (A) a, sin A, sin B

$\Rightarrow$ Two angles are given

$\Rightarrow$ Third angle is also given &

if one side is also given

$\Rightarrow$ A unique Δ is possible



(B) a, b, c

$\Rightarrow$ A unique Δ is possible

(C) a, sin B, R

$\Rightarrow$ b can be calculate

$\Rightarrow$ A unique triangle is possible

(D) a, sin A, R

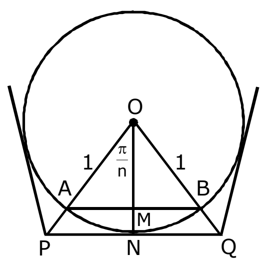
$$\frac{a}{\sin A} = 2R$$

$\Rightarrow$ a & sin A can have different values (not unique) for single value of R

$\Rightarrow$ Δ is not unique

Sol.6 $AM = \sin\frac{\pi}{n}$

$OM = \cos\frac{\pi}{n}$



$$\Delta_{OAB} = \frac{1}{2}\left(2\sin\frac{\pi}{n}\right)\left(\cos\frac{\pi}{n}\right)$$

$$= \sin\frac{\pi}{n}\cos\frac{\pi}{n}$$

$I_n = n\,\Delta_{OAB}$

$$I_n = n\sin\frac{\pi}{n}\cos\frac{\pi}{n}$$

$$\frac{PN}{ON} = \tan\frac{\pi}{n} \Rightarrow PN = \tan\frac{\pi}{n}$$

$$\Rightarrow PQ = 2\tan\frac{\pi}{n}$$

$$\Delta_{OPQ} = \frac{1}{2}\left(2\tan\frac{\pi}{n}\right).1 \Rightarrow \Delta_{OPQ} = \tan\frac{\pi}{n}$$

$$O_n = n\tan\frac{\pi}{n}$$

$$\text{R.H.S.} = \frac{O_n}{2}\,.\left(1+\sqrt{1-\left(\frac{2I_n}{n}\right)^2}\right)$$

$$= \frac{n\tan\frac{\pi}{n}}{2}\left(1+\sqrt{1-\left(\frac{2n\sin\frac{\pi}{n}\cos\frac{\pi}{n}}{n}\right)^2}\right)$$

$$= \frac{n\tan\frac{\pi}{n}}{2}\left(1+\sqrt{1-\left(\sin\frac{2\pi}{n}\right)^2}\right)$$

$$= \frac{n\tan\frac{\pi}{n}}{2}\left(1+\cos\frac{2\pi}{n}\right)$$

$$= \frac{1}{2}\,n\tan\frac{\pi}{n} \times 2\cos^2\frac{\pi}{n}$$

$$= n\sin\frac{\pi}{n}\cos\frac{\pi}{n}$$

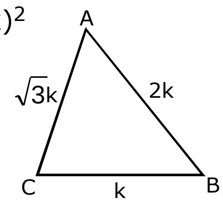
$= I_n =$ L.H.S

Sol.7 D

$a : b : c = 1 : \sqrt{3} : 2$

$\Rightarrow a = k,\ b = \sqrt{3}k,\ c = 2k$

$\because (k)^2 + (\sqrt{3}k)^2 = (2k)^2$



$\Rightarrow a^2 + b^2 = c^2$

$c = 90º$

other angle are 30º & 60º

$\because a < b \quad \Rightarrow A < B$

$A = 30º \qquad B = 60º$

$A : B : C = 30 : 60º : 90º$

$= 1 : 2 : 3$

Sol.8 (a) B

$$\frac{b-c}{a} = \frac{\sin B - \sin C}{\sin A}$$

$$= \frac{2\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)}{2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$\Rightarrow (b-c)\cos\frac{A}{2} = a\sin\left(\frac{B-C}{2}\right)$$

$$\& \ \frac{b+c}{a} = \frac{\sin B + \sin C}{\sin A}$$

$$= \frac{2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$\Rightarrow (b+c)\sin\frac{A}{2} = a\cos\left(\frac{B-C}{2}\right)$$

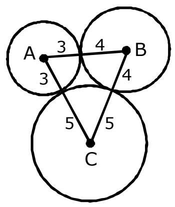
(b) a = 9, b = 8, c = 7

$S = \frac{24}{2} \Rightarrow S = 12$

$\Delta = \sqrt{12(12-9)12-8)(12-7)}$

$= \sqrt{12 \cdot 3 \cdot 4 \cdot 5}$

$= 6\sqrt{20} = 12\sqrt{5}$



Intersection of common tangents is P

$r = \frac{\Delta}{s} = \frac{12\sqrt{5}}{12}$

$\Rightarrow r = \sqrt{5}$

Sol.9 (a) C

b = c & A = 120º & $r = \sqrt{3}$

$$R = \frac{r}{4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}} = \frac{\sqrt{3}}{4.\frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2}$$

$$= \frac{8}{2(\sqrt{3}-1)^2} = \frac{4}{4-2\sqrt{3}} = \frac{2}{2-\sqrt{3}}$$

$\Rightarrow R = 2(2 + \sqrt{3})$

b = c = 2R sin 30º

$= 2.2(2 + \sqrt{3})\ \frac{1}{2}$

$\Delta = \frac{1}{2}b^2 \sin 120º = \frac{4(2+\sqrt{3})^2}{2}\ \frac{\sqrt{3}}{2}$

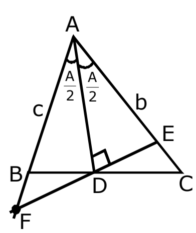
$= (7 + 4\sqrt{3})\ \sqrt{3}$

$= (12 + 7\sqrt{3})$

(b) A,B,C,D

AE = AF

$\Rightarrow \Delta$AEF is Isosceles D



$AD = \frac{2bc}{b+c}\cos\frac{A}{2} \rightarrow (B)$

$\frac{AD}{AE} = \cos\frac{A}{2} \Rightarrow AE = \frac{AD}{\cos\frac{A}{2}}$

$\Rightarrow AE = \frac{2bc}{b+c} \rightarrow (A)$

DE = DF ; $\frac{DE}{AE} = \sin\frac{A}{2}$

EF = 2DE

$EF = \frac{4bc}{b+c}\sin\frac{A}{2} \rightarrow (C)$

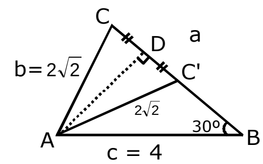
Sol.10 AC = AC' = $2\sqrt{2}$

B = 30º

AB = 4

CD = C'D

sine rule



$\frac{b}{\sin B} = \frac{c}{\sin C}$ {ΔABC or ΔABC'

$\frac{2\sqrt{2}}{\sin 30º} = \frac{4}{\sin C}$

$\Rightarrow 4\sqrt{2} = \frac{4}{\sin C} \Rightarrow \sin C = \frac{1}{\sqrt{2}}$

$\Rightarrow c = \frac{\pi}{4}$ or $c = \frac{3\pi}{4}$

$\because \angle AC'C + \angle AC'B = \pi$

$c = \frac{\pi}{4}$ in ΔABC $\Rightarrow$ A = 105º

& $c' = \frac{3\pi}{4}$ in ΔABC' $\Rightarrow$ A = 15º

Area of $\Delta_{ABC} = \frac{1}{2}4.2\sqrt{2}\ \sin 105º$

Area of $\Delta_{ABC} = 4\sqrt{2}\ \cos 15º$

& Area of $\Delta_{ABC'} = 4\sqrt{2}\ \sin 15º$

Area of Δ_{ABC} – Area of $\Delta_{ABC'}$

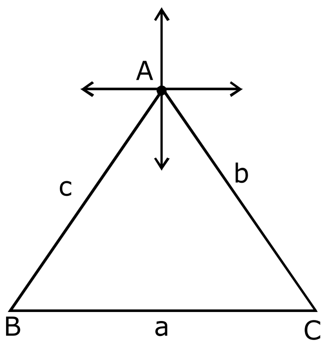
$= 4\sqrt{2}$ (cos 15º – sin 15º)

$$= 4\sqrt{2}\left[\frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}\right]$$

$$= \frac{4\sqrt{2}}{2\sqrt{2}}\ [\sqrt{3} + 1 - \sqrt{3} + 1]$$

= 2.2 = 4

Sol.11 B,C



$\cos B + \cos C = 4\sin^2 \frac{A}{2}$

$\Rightarrow 2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right) = 4\sin^2\left(\frac{A}{2}\right)$

$\Rightarrow \cos\left(\frac{B-C}{2}\right) = 2\sin\frac{A}{2}$

$\Rightarrow \cos\left(\frac{B-C}{2}\right)\cos\frac{A}{2} = 2\sin\frac{A}{2}\cos\frac{A}{2}$

$\Rightarrow \frac{1}{2}\left(2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)\right) = \sin A$

$\Rightarrow \frac{1}{2}[\sin B + \sin C] = \sin A$

$\Rightarrow \sin B + \sin C = \sin A$

$\Rightarrow b + c = 2a$

Locus of a point A about two point B & C (which has fix distance 'a' between them that)

$AB + AC = 2BC \Rightarrow AB + BC =$ consant

then locus of A is ellipse.

Sol.12 D

$2B = A + C$

$A + B + C = 180^o$

$\Rightarrow 3B = 180^o \qquad \Rightarrow B = 60^o$

$\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$

$= \frac{a2(\sin c)\cos c}{c} + \frac{c2(\sin A)\cos A}{a}$

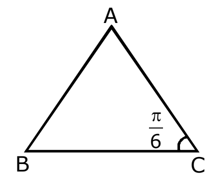
$\because \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$= \frac{2\sin B}{b}[a\cos C + c\cos A]$

$= \frac{2\sin B}{b} \times b$ [by projection rule]

$= 2\sin B = 2\sin 60^o = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$

Sol.13 B



$a = x^2 + x + 1$

$b = x^2 - 1$

$c = 2x + 1$

$a > 0 \quad \forall x \in R$

$b > 0 \quad x \in (-\infty, -1) \cup (1, \infty)$

$c > 0 \quad x \in (-\frac{1}{2}, \infty)$

$\Rightarrow x \in (1, \infty)$ or $x > 1 \qquad \therefore a > b$

$\cos A = \frac{(x^2-1)^2 + (2x+1)^2 - (x^2+x+1)^2}{2(x^2-1)(2x-1)}$

$= \frac{(1+2x-x^2-2x^3)}{2(2x^3+x^2-2x-1)} = \frac{-1}{2}$

$\Rightarrow A = 120^o, \; B = 30^o$

$B = C \; b = C$

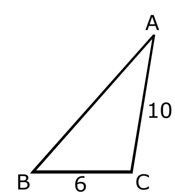
$x^2 - 1 = 2x - 1$

$x^2 - 2x - 2 = 0$

$x = \frac{2 \pm \sqrt{4+8}}{2} = 1 \pm \sqrt{3}$

$\because x > 1 \qquad \therefore x = 1 + \sqrt{3} \quad \{x \neq 1 - \sqrt{3}$

Sol.14 $\angle ACB =$ obtuse



$\Delta = 15\sqrt{3} = \frac{1}{2}ab\sin C$

$\Rightarrow 15\sqrt{3} = \frac{1}{2}6.10\sin C$

$\Rightarrow \sin C = \frac{\sqrt{3}}{2} \Rightarrow C = 120^o$

$\because$ C is obtuse

$\therefore \cos C = \frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow -\frac{1}{2} = \frac{36+100-c^2}{2\cdot 6\cdot 10} \Rightarrow -60 = 136 - c^2$

$\Rightarrow c^2 = 196 \Rightarrow c = 14 \quad \because \{c \neq -14$

$s = \frac{10+6+14}{2} = 15 \Rightarrow s = 15$

$r^2 = \frac{\Delta^2}{s^2} = \frac{15^2.3}{15^2}$

$r^2 = 3$